TE/Insem.-127

TE E&TC 2012 course In Semester Examination
Digital Signal Processing(2012)(304182) Solution

Q.1 a. **600 Hz** 1Mark

$x(n) = \cos(150\pi n/400) + 2\sin(300\pi n/400) - 4\cos(600\pi n/400)$

$= \mathbf{\cos(2\pi\ 3/16\ n) + 2\sin(2\pi\ 3/8\ n) - 4\cos(2\pi\ 3/4\ n)}$ 2 Mark

Yes 1 Mark

The last frequency of 300 Hz, equivalent to a DT frequency of

3/4, is aliased into (1-3/4) 400 Hz = **100 Hz** 2 Mark

b. $\overline{W_4} = \begin{vmatrix} W_4{}^0 W_4{}^0 W_4{}^0 W_4{}^0 \\ W_4{}^0 W_4{}^1 W_4{}^2 W_4{}^3 \\ W_4{}^0 W_4{}^2 W_4{}^4 W_4{}^6 \\ W_4{}^0 W_4{}^3 W_4{}^6 W_4{}^9 \end{vmatrix} = \begin{vmatrix} W_4{}^0 W_4{}^0 W_4{}^0 W_4{}^0 \\ W_4{}^0 W_4{}^1 W_4{}^2 W_4{}^3 \\ W_4{}^0 W_4{}^2 W_4{}^0 W_4{}^2 \\ W_4{}^0 W_4{}^3 W_4{}^2 W_4{}^1 \end{vmatrix}$ 1 Mark

$= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{vmatrix}$ 1 Mark

Now $\overline{W_4{}^{*T}} = = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{vmatrix}$ 1 Mark

Hence $\overline{W_4} W_4{}^{*T} = = 4 \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 4\ \mathrm{U}$ 1 Mark

Q.2. a.

Statement	1 Marks
Explanation	2 Marks
Input signal conditioning (anti-aliasing filter)	1 Marks

b.

i)	a1 . b1 = 0	orthogonal	1 Mark
ii)	a2 . b2 = 23	not orthogonal	1 Mark
iii)	a3 . b3 = 25	not orthogonal	1 Mark
iv)	a4 . b4 = 0	orthogonal	1 Mark

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a1ortho = a1 / ||a1|| = [-2 1 3 -1 1] / 4 ½ Mark
b1ortho = b1 / ||b1|| = [4 -1 0 -1 8]/ √82 ½ Mark

a4ortho = a4 / ||a4|| = [1 3 -3 2 -1] / √24 ½ Mark
b4ortho = b4 / ||b4|| = [-4 1 -3 -2 4] /√46 ½ Mark

Q.3. a. x1(n) = [1 2 1 -2], X1 = [2.0, 0 - 4.0i, 2.0, 0 + 4.0i] 2 Marks
x2(n) = [3-2 1 -3], X2 = [-1.0, 2.0 - 1.0i,9.0, 2.0 + 1.0i] 2 Marks

X1 X2 = [-2, -4 -8i, 18, -4 +8i]
IDFT (X1 X2) = [2, -1, 6, -9] 2 Marks

Verification by graphical method 2 Marks

b. Obtaining DFT from DTFT 1 Mark
Why DFT is used 1 Mark

Q.4. a. DCF formula 2 Marks
x(n) = [1,3,5,7]
C(k) = [8.00, -4.4609, 0.0, -0.3170] 4 Marks

b. Comparision 4 Marks

Q.5 a. $y(n) = 0.7y(n-1) - 0.12y(n-2) + x(n-1) + x(n-2)$

$$Y(z) = 0.7z^{-1}Y(z) - 0.12z^{-2}Y(z) + z^{-1}X(z) + z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}(1+z^{-1})}{1 - 0.7z^{-1} + 0.12z^{-2}} = \frac{z+1}{z^2 - 0.7z + 0.12} = \frac{z+1}{(z-0.4)(z-0.3)}$$

2 Marks

The poles of the system function are $z_1 = 0.4$; $z_2 = 0.3$ and the region of convergence is $|z| > 0.4$.

The poles are lying inside the unit circle. So, the system is stable. 1 Mark

For the input $x(n) = nu(n)$

$$X(z) = \frac{z}{(z-1)^2}$$

$$\frac{Y(z)}{X(z)} = \frac{z+1}{(z-0.4)(z-0.3)}$$ 2 Marks

$$Y(z) = \frac{(z+1)z}{(z-1)^2(z-0.4)(z-0.3)}$$

$$\frac{Y(z)}{z} = \frac{z+1}{(z-1)^2(z-0.4)(z-0.3)}$$

$$= \frac{c_1}{z-0.4} + \frac{c_2}{z-0.3} + \frac{c_3}{z-1} + \frac{c_4}{(z-1)^2}$$

$$= \frac{38.89}{z-0.4} - \frac{26.53}{z-0.3} - \frac{12.36}{z-1} + \frac{4.76}{(z-1)^2}$$

$$Y(z) = \frac{38.89z}{z-0.4} - \frac{26.53z}{z-0.3} - \frac{12.36z}{z-1} + \frac{4.76z}{(z-1)^2}$$

$$y(n) = 38.89(0.4)^n u(n) - 26.53(0.3)^n u(n) - 12.36u(n) + 4.76nu(n)$$

$y(n) = 38.89(0.4)^n u(n) - 26.53(0.3)^n u(n) - 12.36u(n) + 4.76nu(n)$

2 Marks

Q.5. b. $x(n) = (0.5)^n u(n) + (-0.2)^n u(n-3)$

$= (0.5)^n u(n) + (-0.2)^{n-3} (-0.2)^3 u(n-3)$ 1 Mark

$= (0.5)^n u(n) - 0.008\,(-0.2)^{n-3} u(n-3)$

$\therefore X(z) = \frac{1}{1-0.5z^{-1}} - 0.008\,\frac{z^{-3}}{1+0.2z^{-1}}$ ROC: $|z| > 0.5$

2 Marks

Q.6. a. Given $X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$. Multiply numerator and denominator by z^3 2 Marks

$$X(z) = \frac{z^3}{(z-2)(z-1)^2}$$

Dividing the above result by z we have

$$\frac{X(z)}{z} = \frac{z^2}{(z-2)(z-1)^2} = \frac{C_1}{z-2} + \frac{C_2}{z-1} + \frac{C_3}{(z-1)^2}$$ 1 Mark

$$C_1 = (z-2)\frac{X(z)}{z}\Big|_{z=2} = \cancel{(z-2)}\frac{z^2}{\cancel{(z-2)}(z-1)^2}\Big|_{z=2} = 4$$

C_2 and C_3 can be found by using Eq. (2.63) and Eq. (2.64) respectively

$$C_2 = \frac{1}{1!}\frac{d}{dz}\left[(z-1)^2\frac{X(z)}{z}\right]\Big|_{z=1}$$

$$= \frac{d}{dz}\left[\frac{\cancel{(z-1)^2}z^2}{(z-2)\cancel{(z-1)^2}}\right]\Big|_{z=1} = \frac{(z-2)2z-z^2}{(z-2)^2}\Big|_{z=1} = -3$$

$$C_3 = (z-1)^2\frac{X(z)}{z}\Big|_{z=1} = \cancel{(z-1)^2}\frac{z^2}{(z-2)\cancel{(z-1)^2}}\Big|_{z=1} = -1$$ 2 Marks

Substituting C_1, C_2 and C_3 values

$$\frac{X(z)}{z} = \frac{4}{z-2} - \frac{3}{z-1} - \frac{1}{(z-1)^2} \quad \text{or} \quad X(z) = \frac{4z}{z-2} - \frac{3z}{z-1} - \frac{z}{(z-1)^2}$$

1 Mark

$$x(n) = 4(2)^n u(n) - 3u(n) - nu(n)$$

Q.6. b. Statement 1 Mark

Proof 3 Marks